

Exercise 81

A Cepheid variable star is a star whose brightness alternately increases and decreases. The most easily visible such star is Delta Cephei, for which the interval between times of maximum brightness is 5.4 days. The average brightness of this star is 4.0 and its brightness changes by ± 0.35 . In view of these data, the brightness of Delta Cephei at time t , where t is measured in days, has been modeled by the function

$$B(t) = 4.0 + 0.35 \sin\left(\frac{2\pi t}{5.4}\right)$$

- (a) Find the rate of change of the brightness after t days.
- (b) Find, correct to two decimal places, the rate of increase after one day.

Solution

The rate of change of the brightness is given by the derivative of $B(t)$.

$$\begin{aligned} B'(t) &= \frac{dB}{dt} \\ &= \frac{d}{dt} \left[4.0 + 0.35 \sin\left(\frac{2\pi t}{5.4}\right) \right] \\ &= \frac{d}{dt}(4.0) + \frac{d}{dt} \left[0.35 \sin\left(\frac{2\pi t}{5.4}\right) \right] \\ &= (0) + 0.35 \frac{d}{dt} \left[\sin\left(\frac{2\pi t}{5.4}\right) \right] \\ &= 0.35 \left[\cos\left(\frac{2\pi t}{5.4}\right) \cdot \frac{d}{dt} \left(\frac{2\pi t}{5.4}\right) \right] \\ &= 0.35 \left[\cos\left(\frac{2\pi t}{5.4}\right) \cdot \left(\frac{2\pi}{5.4}\right) \right] \\ &= \frac{7\pi}{54} \cos\left(\frac{2\pi t}{5.4}\right) \end{aligned}$$

Evaluate it at $t = 1$.

$$B'(1) = \frac{7\pi}{54} \cos\left(\frac{2\pi}{5.4}\right) \approx 0.16 \frac{\text{brightness unit}}{\text{day}}$$